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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

48. Proposed by P. H. PHILBRICK, C. E., Pineville, Louisiana.

A, B, C, D, and E play with dice, each throwing three, three successive times, for a stake a. A, B, and C throw; C throwing the highest, 52. What is his expectation?

I. Solution by the PROPOSER.

If D or E or both throw 52, C gets but a part of the stake. If D or E or both throw 53 or 54, C gets none of the stake.

$52=18+18+16=18+17+17$. $53=18+18+17$. $54=18+18+18$.

The chance of throwing 16 at a single throw is $\frac{6}{216}$.

The chance of throwing 17 at a single throw is $\frac{3}{216}$.

The chance of throwing 18 at a single throw is $\frac{1}{216}$.

Hence since D may throw 16 (or 18) at any one of the three throws, his chance of throwing 52 at three throws is $3(\frac{1}{216}\times\frac{1}{216}\times\frac{6}{216})+3(\frac{1}{216}\times\frac{1}{216}\times\frac{3}{216})=p_1$ say. E has the same chance of reaching the same result. The chance that D (or E) will not throw 52 is $(1-p_1)$; and the chance that D or E will throw 52 and the others not is $p_1(1-p_1)$, in which case the expectation is $p_1(1-p_1)\frac{1}{2}a$.

The chance that D, and E also, will throw 52 is p_1^2, in which case their joint expectation is $p_1^2\frac{2}{3}a$. Hence the expectation of D or E or of both, coming from throwing 52 is, $2p_1(1-p_1)\frac{1}{2}a+p_1^2\frac{2}{3}a=p_1(3-p_1)\frac{1}{3}a$.

The chance of D or E throwing 53 is, $3(\frac{1}{216}\times\frac{1}{216}\times\frac{3}{216})=p_2$; and the chance that one or both will throw 53 is, $2p_2(1-p_2)+p_2^2=p_2(2-p_2$; and their joint expectation is, $p_2(2-p_2)a$.

The chance that D or E will throw 54 is $(\frac{1}{216}+\frac{1}{216}+\frac{1}{216})=p_3)$; and the chance that one or both will throw 54 is, $2p_3(1-p_3)+p_3^2=p_3(2-p_3)$; and their joint expectation is, $p_3(2-p_3)a$. Hence C's expectation is,

$$\{1-\tfrac{1}{3}[p_1(3-p_1)]-p_2(2-p_2)-p_3(2-p_3)\}a=(1+325p_3^2-47p_3)a.$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas, and J. SCHEFFER, A. M., Hagerstown, Maryland.

D and E may each throw 52, 53, or 54.

52 can be thrown as follows: (6, 6, 6), (6, 6, 5), (6, 6, 5); (6, 6, 6), (6, 6, 6), (6, 6, 4).

53 can be thrown as follows: (6, 6, 6), (6, 6, 6), (6, 6, 5).

54 can be thrown as follows: (6, 6, 6), (6, 6, 6), (6, 6, 6).

D's chance of throwing 52, 53, or 54 is,

$$p=\frac{9}{(216)^3}+\frac{3}{216^3}+\frac{3}{216^3}+\frac{1}{216^3}=\frac{16}{(216)^3}=\frac{2^4}{6^9}.$$

$1-p=1-2^4/6^9=\frac{10077680}{10077696}=P$, chance that D will throw less than 52.
P^2=chance that D and E will both throw less.
$\therefore$ aP^2=C's expectation on the supposition that C wins and no ties.

49. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

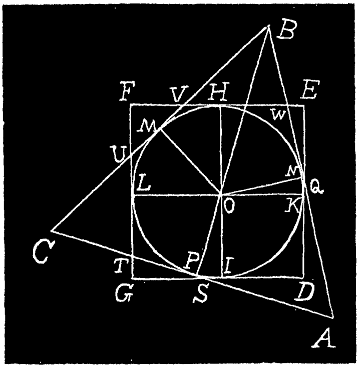
A square whose side is $2a$ and an equilateral triangle whose altitude is $3a$ are fastened together at their centers, but otherwise free to move. If they are thrown on a floor at random, what is the average area common to both ?

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and the PROPOSER.

In the figure let O be the common center of the square and the triangle. Then $OK=ON=OH=OM=OL=OP=OI=a$.

Let the triangle $KON=2\theta$.

Then $\angle NOH=\frac{1}{2}\pi-2\theta$, $\angle HOM=\frac{2}{3}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{6}\pi+2\theta$, $\angle MOL=\frac{1}{3}\pi-(\frac{1}{6}\pi+2\theta)=\frac{1}{3}\pi-2\theta$, $\angle LOP=\frac{2}{3}\pi-(\frac{1}{3}\pi-2\theta)=\frac{1}{3}\pi+2\theta$, and $\angle POI=\frac{1}{2}\pi-(\frac{1}{3}\pi+2\theta)=\frac{1}{6}\pi-2\theta$.



Area of surface, $KONQ$, $=a^2\tan\theta$;
area of surface, $NOHW$, $=a^2\tan(\frac{1}{4}\pi-\theta)$;
area of surface, $HOMV$, $=a^2\tan(\frac{1}{12}\pi+\theta)$;
area of surface, $MOLU$, $=a^2\tan(\frac{1}{6}\pi-\theta)$;
area of surface, $KOPT$, $=a^2\tan(\frac{1}{6}\pi+\theta)$;
area of surface, $POIS$, $=a^2\tan(\frac{1}{12}\pi-\theta)$;
area of square, $OKDI$, $=a^2$.

Hence the area common to the square and triangle is

$$S=a^2[1+\tan\theta+\tan(\tfrac{1}{4}\pi-\theta)+\tan(\tfrac{1}{12}\pi+\theta)+\tan(\tfrac{1}{6}\pi-\theta)+\tan(\tfrac{1}{6}\pi+\theta)+\tan(\tfrac{1}{12}\pi-\theta)].$$

The positions for $\theta>\frac{1}{12}\pi$ are exact repetitions of those for $\theta<\frac{1}{12}\pi$. Hence the required average area is

$$\Delta=\int_0^{\frac{1}{12}\pi}Sd\theta\div\int_0^{\frac{1}{12}\pi}d\theta=\frac{12a^2}{\pi}\int_0^{\frac{1}{12}\pi}\Big[1+\tan\theta+\tan(\tfrac{1}{4}\pi-\theta)+\tan(\tfrac{1}{12}\pi+\theta)+$$

$$\tan(\tfrac{1}{6}\pi-\theta)+\tan(\tfrac{1}{6}\pi+\theta)+\tan(\tfrac{1}{12}\pi-\theta)\Big]d\theta=a^2\Big[1+\frac{12}{\pi}\log_e 2\Big].$$

This problem was also solved in a very excellent manner by G. B. M. Zerr.

50. Proposed by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Find (1), the average length of all straight lines having a given direction, between 0 and a; (2), the average length of chords drawn from one extremity of the diameter a of a semi-circle to all points in the semi-circumference; and (3), find the average area of all triangles formed by a straight line of constant length a sliding between two straight lines at right angles.